PG – 376

First Semester M.Sc. Degree Examination, Jan./Feb. 2014
(Semester Scheme) (N.S.)
MATHEMATICS
M – 102 : Real Analysis

Time : 3 Hours Max. Marks : 80

Instructions *: 1) Answer **any five** full questions choosing atleast **one** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) Show that $f(x) = -x \in R[-C,o]$. **4**

   b) If $f \in R[\alpha]$ on [a, b], then prove that $-f \in R[\alpha]$ on [a, b]. **6**

   c) If $f \in R[\alpha]$ on [a, b], then prove that

   $$\underline{\int_a^b} f d\alpha = \overline{\int_a^b} f d\alpha = \int_a^b f d\alpha = \lambda[\alpha(b) - \alpha(a)], \text{ where } \lambda \in [m, M].$$ **6**

2. a) If f is continuous on [a, b] and α is monotonically increasing function on [a, b], show that $f \in R[\alpha]$. **6**

   b) If f(x) is continuous on [a, b] and α is monotonic on [a, b], prove that

   $$\int_a^b f d\alpha = f(b)\alpha(b) - f(a)\alpha(a) - \alpha(\xi)[f(b) - f(a)]$$

   Where $\xi \in (a,b)$. **6**

   c) Evaluate : $\int_0^5 x^2 d\{[x] - x\}$. **4**

P.T.O.

3. a) Consider two functions $\beta_1(x)$ and $\beta_2(x)$ as follows :

$$\beta_1(x) = \begin{cases} 0, & \text{when } x \le 0 \\ 1, & \text{when } x > 0, \end{cases}$$

$$\beta_2(x) = \begin{cases} 0, & \text{when } x < 0 \\ 1, & \text{when } x \ge 0 \end{cases}$$

Verify, whether $\beta_1(x) \in R[\beta_2(x)]$ on [–1, 1]. **8**

b) State and prove fundamental theorem of integral calculus. **4**

c) Given two functions f and g of bounded variation on [a, b], show that f + g and f.g are also bounded variation. **4**

PART – B

4. a) If $\{f_n(x)\}$ is a sequence of functions defined on [a, b], then prove that $\{f_n(x)\}$ converges uniformly on [a, b] iff $\in > 0$, ∃m, P ∈ N/ for a given

$$|f_{n+P}(x) - f_n(x)| < \in, \forall\, n \ge m, P \ge 1, \forall\, x \in [a, b].$$ **6**

b) Show that $\{\tan^{-1}(nx)\}$ is uniformly convergent on [a, b]. **4**

c) Suppose $f_{n \to 7}$ uniformly on [a, b] and if $x_0 \in [a, b]$ such that

$\lim f_n(x) = A_n$, n = 1, 2, 3,...... $x \to x_0$

Then prove that

i) A_n converges

ii) $$\lim_{x \to x_0} \ \operatorname{lt}_{n \to \infty} f_n(x) = \operatorname{lt}_{n \to \infty} \ \operatorname{lt}_{x \to x_0} f_n(x).$$ **6**

5. a) Let $\{f_n(x)\}$ be a sequence of differentiable functions on [a, b] and let it converge for some point $x_0 \in [a, b]$. If the sequence $\{f_n^1(x)\}$ is uniformly convergent to F(x) on [a, b], then prove that $\{f_n(x)\}$ is uniformly convergent to f(x) on [a, b]. Also prove that $f^1(x) = F(x), \forall\ x \in [a, b]$. **10**

b) Let $\{f_n(x)\}$ be a sequence of functions uniformly convergent to f(x) on [a, b] and each $f_n(x) \in R[a, b]$ then prove that

$f(x) \in R[a, b]$.

Also,

$$\underset{n \to \infty}{\mu} \int_a^x f_n(t)\, dt = \int_a^x f(t)\, dt \ \forall\ x \in [a, b].$$ **6**

6. a) State and prove the Hiene-Borel theorem. **8**

b) Define a K-cell. Prove that every K-cell is compact. **8**

PART – C

7. a) Suppose f maps an open set $E \subset IR^n$ into IR^m, and f is differentiable at a point $x \in E$.Then the partial derivatives $(D_j f_i)(x)$ exist, and

$$f^1(x)ej = \sum_{i=1}^{m} (D_j f_i)(x)(u_i), (1 \le j \le n).$$ **5**

b) If $T \in L(IR^n, IR^m)$, then $||T|| < \infty$ and T is uniformly continuous mapping of IR^n onto IIR^m. **5**

c) If $\phi : X \to X$ is a contraction on a complete metric space X, then prove that ϕ has a unique fixed point. **6**

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